**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676 🡪This Right answer
4. 0.5
5. 0.6987

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| **Solution:** |
|  | Pr(X > 50) = 1 - Pr(X ≤ 50). |
|  | Z = (X - 45)/8.0 |
|  | Thus the question can be answered by using the normal table to find |
|  | Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4% |
|  | Probability that the service manager will not meet his demand = 100-73.4 = 26.6% or 0.2676 |
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1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

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|  | **Solution:**  Probability (X ≤ 44) = Probability ( Z ≤ (44 - 38)/6) = Probability (Z ≤ 1)=84.1345% |
|  | Probability that the employee age will be 44 = 100-84.1345=15.86% |
|  | So the probability of number of employees between 38-44 years of age =  Probability ( X< 44) -0.5=84.1345-0.5= 34.1345% |
|  | Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE. |

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Here we have n = 30 mean = 38 and sd = 6**

ANS: z= (38-36)/6

Probability (x<= 30) = probability (z<=(30-38)/6)

=probability (z<=-1.333)

=9.12%

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|  | Therefore the statement B is TRUE! |
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1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**ANS:** If we take random data for x1 and x2 , the mean will be almost same for 2 *X*1 and *X*1 +

*X*2 but the standard deviation and variance value for x1+x2 will be less compared to 2x1

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1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5 🡪 This is right answer
6. 90.1, 109.9
7. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
8. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**ANS:** Confidence interval for 95% is 1.96

Lower Range= 540-1.96\*225 =98

Upper Range= 540+1.96\*225= 981

= (98, 981)

1. Specify the 5th percentile of profit (in Rupees) for the company

**ANS :** Confidence interval for 90% is 1.64

Lower Range = 540-1.64\*225

Upper Range = 540+1.64\*225

=(171, 909)

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans**: Profit2 has the more probability of making loss in the given year.